



永春高中數學科 階城盃答案卷



班級 215 座號 6 姓名 董德安 第 59 期第 1 大題

(1) $\because$ 圓 O 半徑 $r=1$

$$\therefore \overline{P_1 O_1} = \overline{A O_1} = \overline{B O_1} = \overline{P_2 O_2} \\ = \overline{A O_2} = \overline{B O_2} = \frac{1}{2}$$

$P_i, i \in \{1, 2, 3\}$ 為等分點

$$\Rightarrow \widehat{P_1 P_2} = 120^\circ \Rightarrow \angle P_1 B P_2 = 120^\circ$$

$$\triangle A O_1 B \cong \triangle A O_2 B \text{ (SSS 全等)}$$

$$\Rightarrow \angle O_1 B A = \angle O_2 B A = 60^\circ$$

$$\Rightarrow \angle A O_1 B = \angle A O_2 B = 60^\circ$$

$$a_3 = 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{\pi}{3} - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \sin 60^\circ$$

$$\Rightarrow a_3 = 2 \times \left(\frac{\pi}{24} - \frac{\sqrt{3}}{16} \right)$$

$$\Rightarrow a_3 = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$

$$\Rightarrow a_3 = \frac{2\pi - 3\sqrt{3}}{24}$$

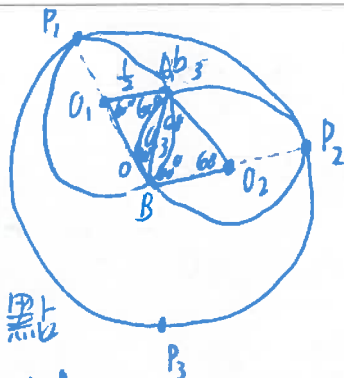
$$b_3 = \text{扇形 } P_1 B P_2 - 2 \times \text{扇形 } A B O + a_3$$

$$= \frac{1}{2} \times 1^2 \times \frac{2\pi}{3} - 2 \times \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \pi + \frac{2\pi - 3\sqrt{3}}{24}$$

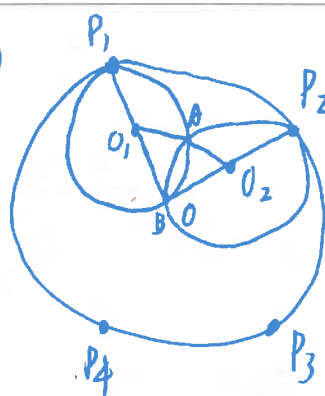
$$= \frac{\pi}{3} - \frac{\pi}{4} + \frac{2\pi - 3\sqrt{3}}{24}$$

$$= \frac{8\pi - 6\pi + 2\pi - 3\sqrt{3}}{24} \Rightarrow \frac{a_3}{b_3} = \frac{2\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}$$

$$= \frac{2\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}$$



(2)



$P_i, i \in \{1, 2, 3, 4\}$ 為等分點

$$\Rightarrow \widehat{P_1 P_2} = 90^\circ \Rightarrow \angle P_1 B P_2 = 90^\circ$$

$$\triangle A O_1 B \cong \triangle A O_2 B \text{ (SSS 全等)}$$

$$\Rightarrow \angle O_1 B A = \angle O_2 B A = 45^\circ$$

$$\Rightarrow \angle A O_1 B = 90^\circ$$

$$a_4 = 2 \times \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \frac{\pi}{2} - \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \sin 45^\circ$$

$$\Rightarrow a_4 = 2 \times \left(\frac{\pi}{16} - \frac{1}{8} \right) \\ = \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

$$b_4 = \text{扇形 } P_1 B P_2 - 2 \times \text{扇形 } A B O + a_4$$

$$= \frac{1}{2} \times 1^2 \times \frac{\pi}{2} - 2 \times \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \pi + \frac{\pi - 2}{8}$$

$$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi - 2}{8}$$

$$= \frac{\pi - 2}{8}$$

$$\Rightarrow \frac{a_4}{b_4} = \frac{\pi - 2}{\pi - 2} = 1$$

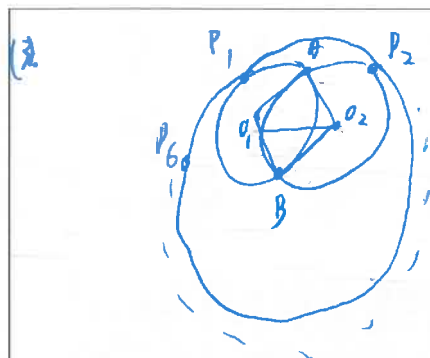
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$P_i, i \in \{1, 2, \dots, 6\}$ 為等分點

$$\Rightarrow \widehat{P_1 P_2} = 60^\circ \Rightarrow \angle P_1 B P_2 = 60^\circ$$

$$\Rightarrow \triangle A O_1 B \cong \triangle A O_2 B \text{ (SSS全等)}$$

$$\Rightarrow \angle O_1 B A = \angle O_2 B A = 30^\circ$$

$$\Rightarrow \angle A O_1 B = 120^\circ$$

$$a_6 = 2 \times \text{Area}_{\triangle A O_1 B}$$

$$= 2 \times \left(\frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \frac{2\pi}{3} - \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \sin 120^\circ \right)$$

$$= 2 \times \left(\frac{\pi}{12} - \frac{\sqrt{3}}{16} \right)$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

$$= \frac{4\pi - 3\sqrt{3}}{24}$$

$$b_6 = \text{Area}_{\triangle P_1 P_2 P_6} - 2 \times \text{Area}_{\triangle A O_1 B} + a_6$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \frac{\pi}{3} - 2 \times \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \pi + \frac{4\pi - 3\sqrt{3}}{24}$$

$$\Rightarrow b_6 = \frac{\pi}{6} - \frac{\pi}{4} + \frac{4\pi - 3\sqrt{3}}{24}$$

$$\Rightarrow b_6 = \frac{4\pi - 6\pi + 4\pi - 3\sqrt{3}}{24}$$

$$\Rightarrow b_6 = \frac{2\pi - 3\sqrt{3}}{24}$$

$$\frac{a_6}{b_6} = \frac{4\pi - 3\sqrt{3}}{2\pi - 3\sqrt{3}}$$

(4) $P_i, i \in \{1, 2, \dots, n\}$ 為等分點

$$\Rightarrow \widehat{P_1 P_2} = \frac{2\pi}{n} \Rightarrow \angle P_1 B P_2 = \frac{2\pi}{n}$$

$$\Rightarrow \triangle A O_1 B \cong \triangle A O_2 B \text{ (SSS全等)}$$

$$\Rightarrow \angle O_1 B A = \angle O_2 B A = \frac{\pi}{n}$$

$$\Rightarrow \angle A O_1 B = \pi - \frac{2\pi}{n} = \frac{(n-2)\pi}{n}$$

$$a_n = 2 \times \text{Area}_{\triangle A O_1 B}$$

$$= 2 \left(\frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \frac{(n-2)\pi}{n} - \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \sin \left(\frac{(n-2)\pi}{n} \right) \right)$$

$$= \frac{(n-2)\pi}{4n} - \frac{\sin \left(\frac{(n-2)\pi}{n} \right)}{4} = \frac{(n-2)\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}{4n}$$

$$b_n = \text{Area}_{\triangle P_1 P_2 P_n} - 2 \times \text{Area}_{\triangle A O_1 B} + a_n$$

$$\Rightarrow b_n = \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \frac{2\pi}{n} - 2 \times \frac{1}{2} \times \left(\frac{1}{2} \right)^2 \times \pi + \frac{(n-2)\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}{4n}$$

$$\Rightarrow b_n = \frac{\pi}{n} - \frac{\pi}{4} + \frac{(n-2)\pi}{4n} - \frac{\sin \left(\frac{(n-2)\pi}{n} \right)}{4}$$

$$\Rightarrow b_n = \frac{2\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}{4n}$$

$$\frac{a_n}{b_n} = \frac{(n-2)\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}{2\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}$$

$$(1) \frac{2\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}$$

$$(2) \frac{1}{2\pi - 3\sqrt{3}}$$

$$(3) \frac{4\pi - 3\sqrt{3}}{2\pi - 3\sqrt{3}}$$

$$(4) \frac{(n-2)\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}{2\pi - n \sin \left(\frac{(n-2)\pi}{n} \right)}$$

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