永春高中數學科　階城盃答案卷

班級 115　座號 5　姓名 周林宏　第 59　期第 一　大題

15

1、① 設小圓半徑為 r，大圓半徑為 $2r$

$a_3=(r\times r\times\frac{1}{6}\pi-r\times r\times\frac{\sqrt{3}}{4})\times 2$

$b_3=2r\times 2r\times\pi\times\frac{1}{3}-r\times r\times\pi+a_3$

$\frac{a_3}{b_3}=\frac{(r^2\times\frac{1}{6}\pi-r^2\times\frac{\sqrt{3}}{4})2}{\frac{4}{3}r^2\pi-r^2\pi+\frac{1}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}=\frac{\frac{\pi}{3}r^2-\frac{\sqrt{3}}{2}r^2}{\frac{2}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}=\frac{2r^2\pi-3\sqrt{3}r^2}{4r^2\pi-3\sqrt{3}r^2}$ —①

已知大圓半徑 $=1$

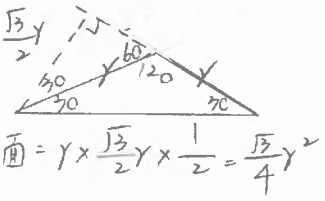
$\therefore r=\frac{1}{2}$ 代入① $\frac{\frac{1}{2}\pi-\sqrt{3}\times\frac{1}{4}}{4\times\frac{1}{4}\pi-3\sqrt{3}\times\frac{1}{4}}=\frac{2\pi-3\sqrt{3}}{4\pi-3\sqrt{3}}$ #

② 設小圓半徑為 r，大圓半徑為 $2r$

$a_4=(r\times r\times\pi\times\frac{1}{4}-r\times r\times\frac{1}{2})\times 2$

$b_4=(2r\times 2r\times\frac{1}{4}\pi-r\times r\times\pi)+a_4$

$\frac{a_4}{b_4}=\frac{a_4}{(r^2\pi-r^2\pi)+a_4}=\frac{a_4}{0+a_4}=1$ #



面 $=r\times\frac{\sqrt{3}}{2}r\times\frac{1}{2}=\frac{\sqrt{3}}{4}r^2$

③ 設小圓半徑為 r，大圓半徑為 $2r$

$a_6=(r\times r\times\pi\times\frac{1}{3}-\frac{\sqrt{3}}{4}r^2)\times 2$

$b_6=(2r\times 2r\times\frac{1}{6}\pi-r\times r\times\pi)+a_6$

$\frac{a_6}{b_6}=\frac{\frac{2}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}{\frac{2}{3}r^2\pi-r^2\pi+\frac{2}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}=\frac{\frac{2}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}{\frac{1}{3}r^2\pi-\frac{\sqrt{3}}{2}r^2}$ —②

已知大圓半徑 $=1$ $\therefore r=\frac{1}{2}$ 代入② $\frac{\frac{1}{6}\pi-\frac{\sqrt{3}}{8}}{\frac{1}{12}\pi-\frac{\sqrt{3}}{8}}=\frac{4\pi-3\sqrt{3}}{2\pi-3\sqrt{3}}$ #

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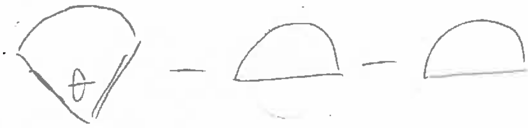
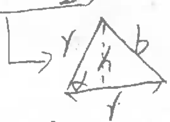
④ $a_n = \Big($  $\Big) \times 2$

$b_n =$  $+ a_n$

設小圓半徑為 r，大圓半徑為 $2r$，$\theta = \frac{360°}{n}$，$\alpha = 180° - \theta = 180° - \frac{360°}{n}$

$a_n = \left(r^2\pi \frac{\alpha}{360°} - \sin\alpha\, r^2 \times \frac{1}{2}\right) \times 2$



$\frac{h}{r} = \sin\alpha$　面積 $\Rightarrow r \times \sin\alpha\, r \times \frac{1}{2}$

$\Rightarrow h = \sin\alpha\, r$　$= \sin\alpha\, r^2 \times \frac{1}{2}$

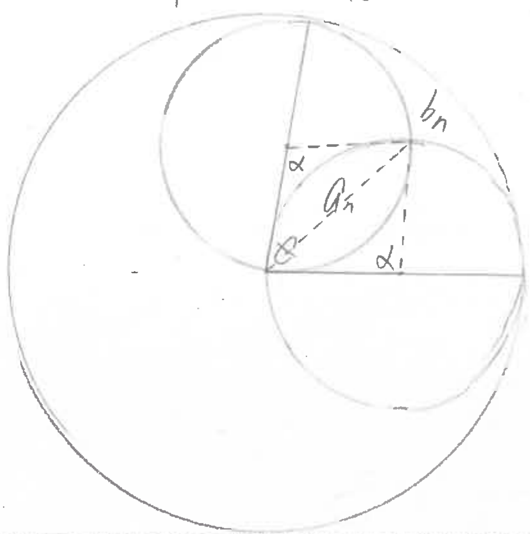
$b_n = 2r \times 2r \times \pi \times \frac{\theta}{360°} - r^2\pi + \left(r^2\pi \frac{\alpha}{360°} - \sin\alpha\, r^2 \frac{1}{2}\right) \times 2$

把 $\theta = \frac{360°}{n}$，$\alpha = 180° - \frac{360°}{n}$ 代入 $\frac{a_n}{b_n}$

$$\Rightarrow \frac{a_n}{b_n} = \frac{r^2\pi \times \frac{180° - \frac{360°}{n}}{180°} - \sin\left(180° - \frac{360°}{n}\right) r^2}{r^2\pi \times \frac{\frac{360°}{n}}{90°} - r^2\pi + r^2\pi \times \frac{180° - \frac{360°}{n}}{180°} - \sin\left(180° - \frac{360°}{n}\right) r^2} = \frac{\left(1 - \frac{2}{n}\right)\pi - \sin\left(180° - \frac{360°}{n}\right)}{\frac{2}{n}\pi - \sin\left(180° - \frac{360°}{n}\right)}$$

\# 且 $n \geq 3$

經整理可得



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