



永春高中數學科 階城盃答案卷

班級 315 座號 1 姓名 陳羽蓮 第 54 期第 二 大題

(1)

甲: $P \rightarrow A \rightarrow P \rightarrow B \rightarrow P \rightarrow C \rightarrow P$ $AP = BP = CP = \frac{5}{3}$
 $\frac{5}{3} \times 2 + \frac{5}{3} \times 2 + \frac{5}{3} \times 2 = 15$

乙: $P \rightarrow D \rightarrow P \rightarrow E \rightarrow P \rightarrow F \rightarrow P$
 $\frac{4}{3} \times 2 + \frac{5}{3} \times 2 + 0 \times 2 = 14$ $\therefore 15 > 14 \therefore$ 乙較快

(2)

甲: $\frac{2}{3}\sqrt{3} \times 2 \times 3 = 4\sqrt{3} + AP = BP = CP = \frac{2}{3} \times 2 \cos 60^\circ = \frac{2}{3}\sqrt{3}$ $\frac{4\sqrt{3}}{\frac{2}{3}\sqrt{3}} = 2$ (倍)

乙: $\frac{1}{3}\sqrt{3} \times 2 \times 3 = 2\sqrt{3} + DP = EP = FP = \frac{1}{3} \times 2 \cos 60^\circ = \frac{1}{3}\sqrt{3}$

(3)

① $\triangle ABC$ 的面積 $= \frac{1}{2} AB \times DP + \frac{1}{2} BC \times EP + \frac{1}{2} CA \times FP$
其中 $AB = BC = CA = 2$
因此 $\triangle ABC = \sqrt{3} = \frac{1}{2} \times 2 (DP + EP + FP) \Rightarrow DP + EP + FP = \sqrt{3}$ 為定值

② $PD : PE : PF = 2 : 3 : 1 \Rightarrow PD = \sqrt{3} \times \frac{2}{6} = \frac{1}{3}\sqrt{3}, PE = \sqrt{3} \times \frac{3}{6} = \frac{1}{2}\sqrt{3}, PF = \sqrt{3} \times \frac{1}{6} = \frac{1}{6}\sqrt{3}$ $+1$
作離 AB, BC, CA 的距離分別為 $\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{6}$ 之水平線, 其交點即為 P

③ 將 $\triangle ABC$ 各點的位置坐標化, $A(2,0), B(1,\sqrt{3}), C(0,0)$,
求出 P, D, E, F 的坐標
得 $P(\frac{7}{6}, \frac{\sqrt{3}}{6}), D(\frac{5}{3}, \frac{\sqrt{3}}{3}), E(\frac{5}{6}, \frac{\sqrt{3}}{2}), F(\frac{7}{6}, 0)$

甲: $2(\sqrt{(\frac{1}{6})^2 + (\frac{\sqrt{3}}{6})^2} + \sqrt{(\frac{1}{6})^2 + (\frac{\sqrt{3}}{6})^2} + \sqrt{(\frac{1}{6})^2 + (\frac{\sqrt{3}}{6})^2})$
 $= 2 \times \frac{\sqrt{1+3} + \sqrt{1+3} + \sqrt{1+3}}{6} = \frac{2\sqrt{4} + 2\sqrt{4} + 2\sqrt{4}}{3} = \frac{6\sqrt{4}}{3} = 4$ $+2$

乙: $2\sqrt{3}$
所求為 $\frac{2\sqrt{4} + 2\sqrt{4} + 2\sqrt{4}}{3} \times \frac{1}{2\sqrt{3}} = \frac{\sqrt{4} + \sqrt{4} + \sqrt{4}}{2\sqrt{3}} = \frac{3\sqrt{4}}{2\sqrt{3}} \approx 6.126$ (倍) $+1$

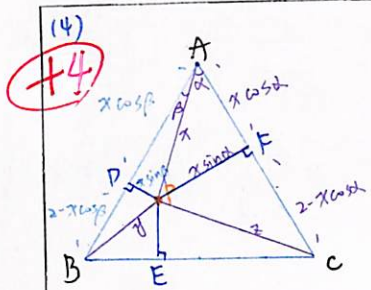
請將本卷對折一次後投入投件箱, 謝謝您的參與!



永春高中數學科 階城盃答案卷



班級 315 座號 1 姓名 陳羽蓮 第 54 期第 二 大題



令 $AP = x$, $BP = y$, $CP = z$, $\angle PAC = \alpha$, $\angle PAB = \beta$, $\alpha + \beta = 60^\circ$

令甲所走的路徑長為 $f(x) \times 2$ + 同一條路經過 $2 = x$

$$\begin{aligned} \text{則 } f(x) &= x + \sqrt{(x \sin \alpha)^2 + (2 - x \cos \alpha)^2} + \sqrt{(x \sin \beta)^2 + (2 - x \cos \beta)^2} \\ &= x + \sqrt{x^2 \sin^2 \alpha + 4 - 4x \cos \alpha + x^2 \cos^2 \alpha} + \sqrt{x^2 \sin^2 \beta + 4 - 4x \cos \beta + x^2 \cos^2 \beta} \\ &= x + \sqrt{x^2 - 4x \cos \alpha + 4} + \sqrt{x^2 - 4x \cos \beta + 4} \\ &= x + \sqrt{x^2 - 4x \cos \alpha + 4} + \sqrt{x^2 - 4(\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha)x + 4} \end{aligned}$$

當 $\alpha = 30^\circ$ 時, $\beta = 60^\circ - 30^\circ = 30^\circ$ 為何不考慮其他角度? (雙變數函數求極值)

$$f_{30}(x) = x + \sqrt{x^2 - 2\sqrt{3}x + 4} + \sqrt{x^2 - 2\sqrt{3}x + 4}$$

$$\begin{aligned} f'_{30}(x) &= [x]' + [2(x^2 - 2\sqrt{3}x + 4)^{\frac{1}{2}}]' \\ &= 1 + 2x \cdot \frac{1}{2} (x^2 - 2\sqrt{3}x + 4)^{-\frac{1}{2}} (2x - 2\sqrt{3}) \\ &= 1 + \frac{x - \sqrt{3}}{\sqrt{x^2 - 2\sqrt{3}x + 4}} \end{aligned}$$

解 $f'_{30}(x) = 0 \Rightarrow x = \frac{2\sqrt{3}}{3} \Rightarrow x = \frac{2\sqrt{3}}{3}$ 時 $f_{30}(x)$ 有最小值 $\rightarrow$ 此時 P 點在重心上

$$\begin{aligned} \text{此時 } f_{30}(x) &= f_{30}(\frac{2\sqrt{3}}{3}) = \frac{2\sqrt{3}}{3} + 2\sqrt{(\frac{2\sqrt{3}}{3} \times \frac{1}{2})^2 + (2 - \frac{2\sqrt{3}}{3} \times \frac{\sqrt{3}}{2})^2} \\ &= \frac{2\sqrt{3}}{3} + 2 \times \frac{2}{3} = 2\sqrt{3} \end{aligned}$$

$\Rightarrow$ 離重心越遠 $f(x)$ 越大, k 越大

$$\min(k) = \frac{2\sqrt{3} \times 2}{2\sqrt{3}} = 2$$

請將本卷對折一次後投入投件箱，謝謝您的參與！